Eur. Phys. J B **19**, 101–106 (2001)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Hypersensitivity to small signals in a stochastic system with multiplicative colored noise

O.V. Gerashchenko[a], S.L. Ginzburg, and M.A. Pustovoit

Petersburg Nuclear Physics Institute, Gatchina Leningrad district 188350, Russia

Received 27 April 2000 and Received in final form 2 November 2000

Abstract. Recently we discovered the phenomenon of hypersensitivity to small time-dependent signals in a simple stochastic system, the Kramers oscillator with multiplicative white noise. In the present work we study, theoretically and experimentally with analog simulations, an influence of noise correlation time on hypersensitivity in a nonlinear oscillator with piecewise-linear current-voltage characteristic and multiplicative colored dichotomous noise. We found that the region of hypersensitive behavior is defined by universal scaling index, whereas the specifics of a particular system reveals itself only in the dependence of the above index on system parameters. The dependence of gain factor on noise correlation time is of bell-shaped (resonant) type.

PACS. 05.45.-a Nonlinear dynamics and nonlinear dynamical systems – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 84.30.-r Electronic circuits

1 Introduction

The role of noise in physical systems in last two decades was subjected a substantial reassessment induced by discovery of numerous interesting phenomena caused by it. Up to now, it is known that in nonlinear systems noise can induce phase transitions [1], complex ordered patterns [2], directed transport of matter [3–7], as well as facilitate transduction of external signal [8–11], waves [12–14] and enhance diffusion [15] in the system.

The next example of constructive role of noise is the noise-induced hypersensitivity to small time-dependent signals recently found by us analytically, numerically and experimentally [16–19] in a nonlinear Kramers oscillator with multiplicative white noise. Under effect of large parametric noise the system was able to amplify an ultrasmall (of the order of, *e.g.,* 10^{-20}) deterministic ac signal up to the value of the order of unity. Such an anomalous sensitivity in the system is a result of on-off intermittency [20–28].

On-off intermittency appears in a dynamical system when it passes through a bifurcation point under effect of external stochastic time-dependent forcing. It attracts a stable interest of investigators due to its several intriguing properties, the most easily observable of which is the specific time behavior of physical quantities: the bursts of large amplitude alternate randomly with the long quiet periods with near-zero amplitude.

On-off intermittency has an important feature of power-law dependence of probability density of burst amplitude [26–28]:

$$F(x) \sim x^{\alpha-1}, \tag{1}$$

where α is the scaling index.

This expression holds in a wide range of amplitudes $A \ll x \ll 1$, where A is the magnitude of small external signal [16] and the system size is set to unity.

For $\alpha < 0$, $|\alpha| \ll 1$ and for vanishingly small external signal ($A \to 0$) we get $F(x) \to \delta(x)$, so the system variable always have near-zero values.

Now, when

$$A > A_0 = \exp(-1/|\alpha|), \tag{2}$$

it appears that the moments of distribution grow up to the order of unity. Because for $|\alpha| \ll 1$ the value of A_0 is exponentially small, practically any physical value of the signal results in a response of the order of unity. We call this phenomena as hypersensitivity. A similar, but more complex situation appears for small positive α.

As it is shown in [16] the hypersensitivity is caused mainly by multiplicative noise. We restricted ourselves there by consideration of white noise only. However, it is intuitively clear that this restriction does not affect the physical nature of the problem, and the phenomenon of hypersensitivity should appear in practically all problems with multiplicative noise. This means that the asymptotical relation (1) is always valid for $A \ll 1$, and we can introduce the parameter α. The only necessary condition for this is the existence of wide parameter range where the scaling index α is small. The condition of its smallness is the condition of existence of hypersensitivity in a particular system. We call α the sensitivity index.

[a] e-mail: gerashch@hep486.pnpi.spb.ru

For Gaussian white noise we found that for adiabatic conditions α is inversely proportional to mean square of the noise. As a result, the condition of hypersensitivity is valid for sufficiently large noise amplitude, *i.e.* hypersensitivity is induced by noise. Further increase of noise amplitude forces the system to leave adiabatic regime, and the signal gain factor decreases, like conventional stochastic resonance.

In the case of colored noise, index of sensitivity depends, as we show below, both on amplitude and on correlation time of the noise. The dependence of signal gain factor on both these parameters is of bell-shaped type. The choice of system with colored noise as an example of versatility of our approach is physically natural, because any real stochastic process has nonvanishing correlation time. We consider the simplest colored noise, the random telegraph noise, when the problem allows an analytical solution. We found that for both noise sources, white and colored, the system behavior is the same, and hypersensitivity region is defined solely by the condition $|\alpha| \ll 1$. Thereby, the phenomenon of hypersensitivity in systems with multiplicative noise is really universal. We should notice, hereinafter, that considered systems with multiplicative noise are the special case of systems with on-off intermittency: their bifurcation parameter changes its sign randomly, and the system behavior for different signs is quite different. One could show also that the scaling (1) is characteristic for all systems with on-off intermittency and therefore the phenomenon of hypersensitivity should appear there.

2 Theory

White noise is known to be an abstraction, since any real physical noise process has a finite correlation time τ, *i.e.*, a real noise is a colored noise. In general theoretical treatment of the problem, we should include white noise limit, when noise amplitude increases together with decreasing of correlation time as $\tau^{-1/2}$. This condition is not valid for $\tau \to 0$. In addition, the finite correlation time should be taken in account for the case when that time is comparable with the period of the external signal. Thus, in this paper we perform a theoretical treatment of the phenomenon of hypersensitivity induced by multiplicative colored noise "from scratch".

We study a general stochastic equation in periodic square-wave field:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x,z) + AE(t),$$
$$E(t+T) = E(t) = \begin{cases} 1, & 0 < t < T/2 \\ -1, & T/2 < t < T. \end{cases} \tag{3}$$

Here $z(t)$ is the random variable with zero mean and with autocorrelation

$$< z(t)z(t') >= \Delta^2 \exp(-\gamma|t-t'|). \tag{4}$$

For telegraph (dichotomous) noise $z(t)$ is as follows:

$$z(t) = \Delta S(t), \qquad S(t) = \pm 1, \tag{5}$$

where $S(t)$ is the random variable that changes its sign with the rate $\gamma/2$. For colored noise the variables $x(t)$ and $z(t)$ define a two-dimensional Markov process, with two-dimensional Fokker-Planck equation (FPE) for probability density $F(x,z,t)$.

For dichotomous noise the variable $z(t)$ can take only two values $\pm\Delta$, and therefore the equation for $F(x,z,t)$ reduces to two equations [1]:

$$\frac{\partial F(x,\pm\Delta,t)}{\partial t} = -\frac{\partial}{\partial x}([f(x,\pm\Delta) + AE(t)]F(x,\pm\Delta,t)) + \frac{\gamma}{2}[F(x,\mp\Delta,t) - F(x\pm\Delta,t)]. \tag{6}$$

Let us discuss now a choice of $f(x,z)$. On-off intermittency means a stochastically modulated bifurcation parameter. With noise z taking only two values $\pm\Delta$, one of them ($z = -\Delta$) must correspond to fixed point $x = 0$, and other ($z = \Delta$) to another fixed point $x = x_1 \neq 0$. In such a case with two competing fixed points on-off intermittency appears for $|\alpha| \ll 1$, together with hypersensitivity. The latter is studied by us with analog simulations, and we use below $f(x,z)$ of the same type as in the analog simulations:

$$f(x,z) = \begin{cases} a(x+x_1); & x < -x_0(z), \\ (\lambda + z)x; & |x| < x_0(z), \\ -a(x-x_1); & x > x_0(z), \end{cases} \tag{7}$$
$$x_0(z) = ax_1/(a+\lambda+z).$$

The function $x_0(z)$ in (7) is chosen in such a way that for $x = x_0(z)$ $f(x,z)$ is a continuous function with a cusp. To make $x_0(z)$ positive we should assume

$$a > \pm\Delta - \lambda. \tag{8}$$

Figure 1 displays $f(x,\pm\Delta)$ for $\Delta > |\lambda|$. We see that for $z = \Delta$ there are two stable fixed points $x = \pm x_1$, and for $z = -\Delta$ there is only one such point $x = 0$. Thereby, in our system the necessary condition for on-off intermittency and hypersensitivity is fulfilled.

The sufficient condition for existence of the latter phenomena is a power-law dependence of $F(x)$ (1) for small $|x| \ll 1$ and small sensitivity index $|\alpha| \ll 1$. To test this condition we should solve equation (3) with $f(x,z)$ from (7) and obtain the asymptotic relation (1). Equation (6) can be solved explicitly only in the adiabatic limit, when the period of the signal T is much greater than the characteristic time of establishing of a stationary distribution. It was shown in [1] that for fixed input signal ($E(t) = 1$) it is possible to find a stationary solution of equation (6). For adiabatic time-dependent signal we can obtain solutions for positive and negative values of input signal separately. For $E(t) = 1$ we obtain for probability density $F(x) = F(x,\Delta) + F(x,-\Delta)$ the following closed expression [1]:

$$F(x) = C\left|\frac{f(x,\Delta) - f(x,-\Delta)}{(f(x,\Delta)+A)(f(x,-\Delta)+A)}\right| \times \exp\left(-\frac{\gamma}{2}\int \mathrm{d}x \left[\frac{1}{f(x,\Delta)+A} + \frac{1}{f(x,-\Delta)+A}\right]\right). \tag{9}$$

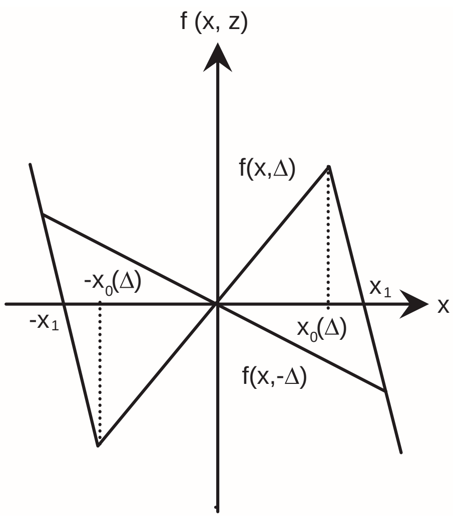

Fig. 1. Function $f(x, z = \pm\Delta)$ in (7).

There are four constants of the same dimensionality in the problem: a, Δ, λ and γ. In further treatment we consider the case

$$\Delta > |\lambda|, \quad a > \Delta + |\lambda|. \tag{10}$$

Thus expression (9) for $\frac{A}{\Delta - A} < x < x_0(\Delta)$ and $x_1 \gg A$ after some standard calculations becomes

$$F(x) = Cx\left(x + \frac{A}{\Delta+\lambda}\right)^{-1-\frac{\gamma}{2(\Delta+\lambda)}} \times \left(x - \frac{A}{\Delta-\lambda}\right)^{-1+\frac{\gamma}{2(\Delta-\lambda)}}. \tag{11}$$

For $x_0(\Delta) < x < x_1$ we get:

$$\begin{aligned} F(x) = {} & Cx^{-1+\frac{\gamma}{2(\Delta-\lambda)}}\,[x_0(\Delta)]^{-\frac{\gamma}{2(\Delta-\lambda)}} \\ & \times (x_1 - x)^{-1+\frac{\gamma}{2a}}\,\frac{\Delta+\lambda}{a}(x_1 - x_0(\Delta))^{-\frac{\gamma}{2a}} \\ & \times \left[\frac{a}{2\Delta}(x_1 - x) + \frac{\Delta-\lambda}{2\Delta}x\right] \\ & F(x) = 0,\ x > x_1. \end{aligned} \tag{12}$$

Here $F(x)$ is defined on the interval (the carrier of probability density function)

$$\left[\frac{A}{\Delta-\lambda}; x_1\right]. \tag{13}$$

For $x = x_0(\Delta)$, $F(x)$ in (11) and (12) must be the same. From (11) and (12) it is easily seen that, as far as $x_0(\Delta) \gg A$:

$$F(x_0) = C[x_0(\Delta)]^{-1+\alpha}, \quad \alpha = \frac{\gamma\lambda}{\Delta^2 - \lambda^2}\,. \tag{14}$$

Further, from (11) we get for $A \ll x < x_0(\Delta)$:

$$F(x) = Cx^{-1+\alpha}, \tag{15}$$

that coincides with (1), with sensitivity index α defined by (14). The condition of hypersensitivity $|\alpha| \ll 1$ gives us:

$$\gamma|\lambda| \ll \Delta^2 - \lambda^2. \tag{16}$$

In the similar manner, for $E(t) = -1$ we get a "mirror image" of (11) and (12). For instance, for $-x_0(\Delta) < x < -\frac{A}{\Delta-\lambda}$ it reads:

$$F(x) = C(-x)\left(-x + \frac{A}{\Delta+\lambda}\right)^{-1-\frac{\gamma}{2(\Delta+\lambda)}} \times \left(-x - \frac{A}{\Delta-\lambda}\right)^{-1+\frac{\gamma}{2(\Delta-\lambda)}}. \tag{17}$$

Note that for $\gamma < 2a$ there is a singularity in (12) also for $x = x_1$.

Let us calculate now the normalization constant C. It is rather complex to obtain it exactly in a general case, but one can easily see that for $|\alpha| \ll 1, \gamma/2a$ the main contribution to normalization constant comes from the region $x \sim A$, *i.e.* from the bottom edge of the carrier of $F(x)$. Therefore a good estimate for C is simply:

$$C^{-1} = B^{-1}\int_A^{x_1} x^{\alpha-1}\mathrm{d}x = B^{-1}\frac{x_1^{\alpha} - A^{\alpha}}{\alpha}, \tag{18}$$

where B is the constant independent on A. Since $|\alpha| \ll 1$, from (18) we obtain:

$$C = B\begin{cases} \alpha, & \alpha > 0, \quad \zeta \gg 1, \\ 1/\ln(1/A), & \qquad\quad\ \ \zeta \ll 1, \\ |\alpha|A^{|\alpha|}, & \alpha < 0, \quad \zeta \gg 1, \end{cases} \qquad \zeta = |\alpha|\ln(1/A). \tag{19}$$

Let us now calculate the moments of distribution in adiabatic (in relation to signal period) approximation. Taking into account that the moments are determined by the region $|x| > A$, we use the expression (15) for $|x| < x_0(\Delta)$ and the similar one for $E(t) = -1$. Then, using (19) we get, for instance, for $\zeta \ll 1$:

$$\begin{gathered} \langle x(t)\rangle \sim E(t)x_1/\ln(1/A), \\ \langle x^2(t)\rangle \sim x_1^2/\ln(1/A), \\ \langle x(t)\rangle^2/\langle x^2(t)\rangle \sim 1/\ln(1/A), \\ I = \frac{\langle x(t)\rangle}{AE(t)} \sim \frac{x_1}{A\ln(1/A)}, \end{gathered} \tag{20}$$

where I is the signal gain factor. The expression for $\zeta \gg 1$ are similar. We see that, with an accuracy up to $\ln(1/A)$ all the moments of $x(t)$ have values of the order of the corresponding power of x_1, and the gain factor $I \sim \frac{x_1}{A} \gg 1$. On the other hand, when taking $A = 0$, all the moments for $\alpha < 0$ vanish, since $F(x) \to \delta(x)$. As can be seen from (19), the crossover occurs at $\xi \sim 1$, *i.e.* for $A \sim A_0$ from (2). The described phenomenon, when the gain factor can reach many orders of magnitude, is called hypersensitivity.

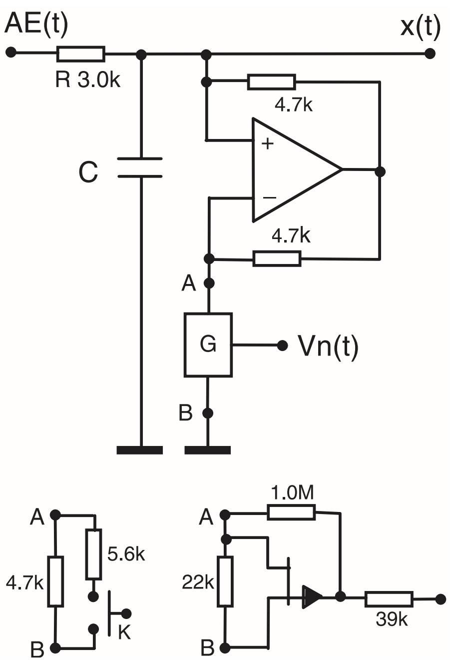

Fig. 2. The electronic circuit used in our analog simulations. Operation amplifier is $\mu A740$, electromagnetic relay and $C = 2.0\ \mu$F, (circuit layout 1), field-effect transistor is $2N5114$ and $C = 0.108\ \mu$F (circuit layout 2).

3 Analog simulations

Figure 2 presents two layouts of electronic circuit used in our analog simulations. Its main blocks are the capacitor and the nonlinear element with conductivity $G(t)$ controlled by voltage $V_n(t)$, connected in parallel. The nonlinear element is designed using an operational amplifier and a commuter (an electromagnetic relay in the layout 1 and a field transistor in the layout 2). The voltage $V_n(t)$ have the form of dichotomous (telegraph) noise. Note that, using a field transistor as a switching element (layout 2), one could apply any noise to the circuit, not only a dichotomous one, since there is a region with linear dependence of conductivity of field transistor channel on controlling voltage on shutter (in [18,19] we used white Gaussian noise).

An input square-wave signal is represented by voltage $AE(t)$ with zero mean, amplitude A and period T, applied through the resistor R. Let us write a Kirchhoff equation for our circuit:

$$\frac{AE(t) - x(t)}{R} = C\frac{\mathrm{d}x}{\mathrm{d}t} + I_1(x,z), \qquad (21)$$

where $x(t)$ is the voltage under study, $C\mathrm{d}x/\mathrm{d}t$ is the current through capacitor, $I_1(x,z)$ is the current through the nonlinear element with fluctuating conductivity, z is the random telegraph signal that modulates the above conductivity.

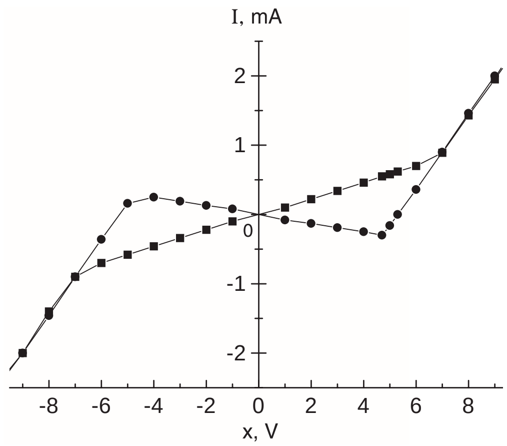

Fig. 3. The static current-voltage characteristics of electronic circuit (Fig. 1, layout 1). Relay is switched by multiplicative noise and modulates conductivity.

From (21) we get:

$$\begin{aligned} RC\frac{\mathrm{d}x}{\mathrm{d}t} &= -RI_1(x,z) - x + AE(t) \\ &= -RI(x,z) + AE(t), \\ I(x,z) &= I_1(x,z) + x/R, \end{aligned} \qquad (22)$$

where $I(x,z)$ is the current-voltage characteristics (CVC) of the circuit. Its exact form, obtained experimentally for two values of telegraph noise z, is represented by Figure 3. Equation (21) is identical to equation (3) (see Figs. 1 and 3) when substituting

$$\frac{t}{RC} \to t, \quad -RI(x,z) \to f(x,z). \qquad (23)$$

The parameters of $f(x,z)$ (7) are:

$$\Delta = 0.27, \quad \lambda = -0.09, \quad a = 1.59, \quad x_1 = 5.29\ \mathrm{V}, \qquad (24)$$

where x is in volts and time is in units $RC = 6 \times 10^{-3}$ s (layout 1) and $RC = 3.24 \times 10^{-4}$ s (layout 2), value γ is in units $(RC)^{-1}$.

Figure 4 shows typical time series $x(t)$ for circuit in layout 1 for amplitude of input square-wave signal $A = 0.05$ V with period $T = 3.3$ s (the dimensionless frequency $\omega_s = 2\pi RC/T = 2\pi RC f_s = 0.0113$) for various values of parameter γ (γ is defined by (4)). We see that a signal of small amplitude A is amplified up to the value of the order of x_1, in accordance with equation (20). Figure 5 displays the spectral density of the output signal for the same values of parameters.

Such a behaviour is characteristic for on-off intermittency, when a system, being in laminar phase ($x \ll x_1$), is excited up to the cutoff ($x \approx x_1$) and returns to laminar phase again soon. It can be seen also that for $\gamma < \omega_s$ output switchings not always follow to switchings of input signal (the adiabaticity is broken) and the fundamental harmonics in output signal spectrum is small. Further increase of γ leads to switchings for each half of signal period and the gain factor increases.

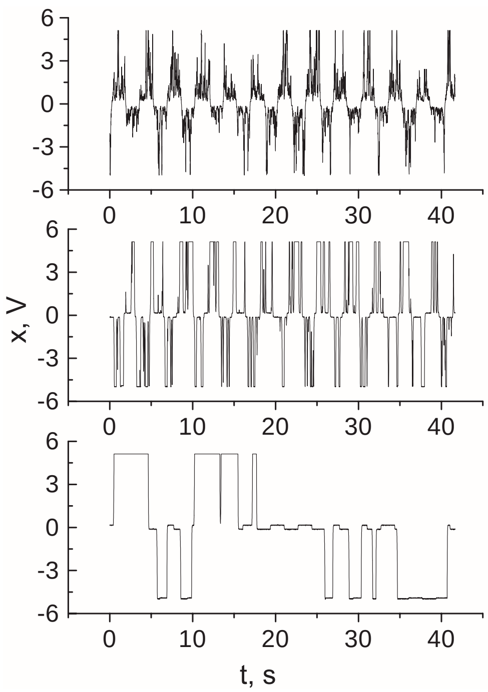

Fig. 4. The output voltage $x(t)$ (circuit layout 1) for input square-wave signal with the amplitude $A = 50$ mV and frequency 0.3 Hz (dimensionless signal frequency $\omega_s = 2\pi RC f_s = 0.0113$) for several values of parameter γ (curves from above): $\gamma = 1.14 \gg \Delta = 0.27$ (the colored noise acts as an "effectively white" one), $\gamma = 0.065,\ 0.0068 \ll \Delta$.

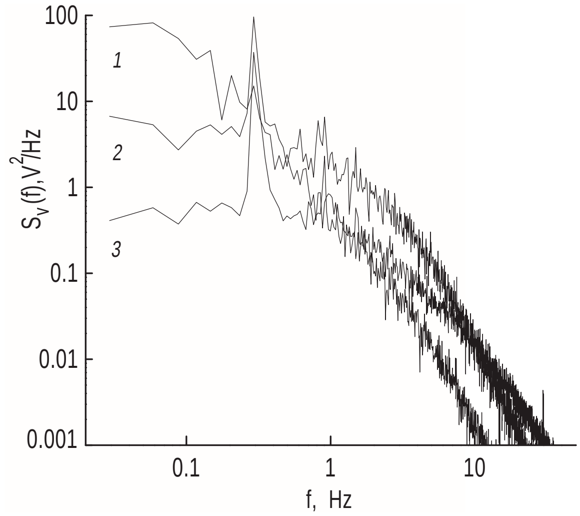

Fig. 5. Power spectral density of output voltage fluctuations for input square-wave signal with the amplitude $A = 50$ mV and frequency 0.3 Hz ($\omega_s = 0.0113$) for values of parameter $\gamma = 0.0068(1),\ 0.065(2),\ 1.14(3)$.

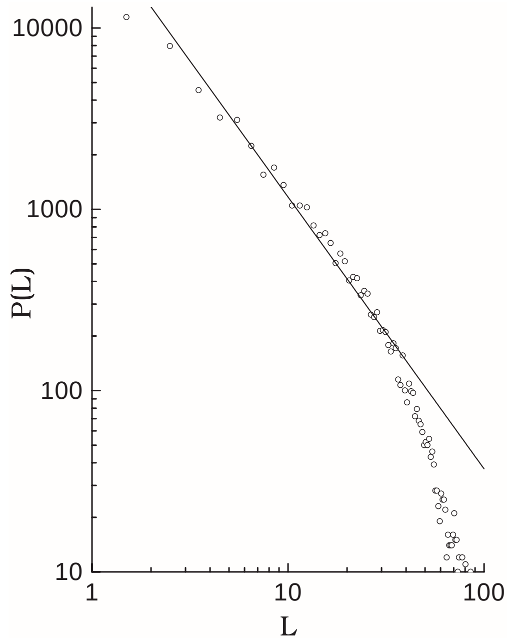

Fig. 6. Distribution of laminar lengths for constant input signal $AE(t) = A = 15$ mV and laminarity threshold $p = 0.1$ for $\gamma = 1.14$. The solid line is drawn according to equation (25).

A common fingerprint of on-off intermittency, together with the power-law dependence of stationary probability density (15), is a scaling behaviour of the distribution of laminar phase lengths $P(L)$ [22–28]:

$$P(L) \sim L^{-3/2}, \tag{25}$$

where L is the length of laminar phase. This dependence is observed in the interval of laminar lengths $1/\gamma \ll L \ll 1/|\lambda|$. Thus the measurement of $P(L)$ gives us another way of determination of parameters γ and λ. We determine the laminar phase lengths experimentally as follows: in the time series $x(t)$ we found a maximum $x_{\max} = x_1 \approx 5$ V, and with predefined threshold p the laminar phase of $x(t)$ is defined by the condition $x(t) < px_{\max}$. Figure 6 demonstrates the dependence $P(L)$ for constant input signal $AE(t) = A = 0.015$ V and laminarity threshold $p = 0.1$. $P(L)$ agrees well with theoretical prediction equation (25). From Figure 6 we obtain $|\lambda| \approx 0.1$ and $\gamma \sim 1$, that complies with the direct measurement of $\gamma \approx 1.14$ from the spectrum of controlling noise, and with the value $\lambda = -0.09$ determined from CVC in Figure 3.

Figure 7 shows the dependence of gain factor from the value of parameter γ. The gain factor is defined as

$$K(A) = \sqrt{S_V(f_s)\Delta f}/A, \tag{26}$$

where $S_V(f_s)$ is the spectral intensity of the fundamental harmonics of output signal ($f_s = 1/T = 0.3$ Hz, $\omega_s = 2\pi f_s RC$), $\Delta f = 0.03$ Hz is the spectral bandwidth. For the circuit 1 we see that for $\gamma < \omega_s$ the adiabatic condition for signal is violated and the gain factor decreases. In the circuit 2 the dimensionless signal frequency is $\omega_s = 6.1 \times 10^{-4} < \gamma$. However for large γ the parameter α is close to unity, according to the expression (14), and the gain decreases again.

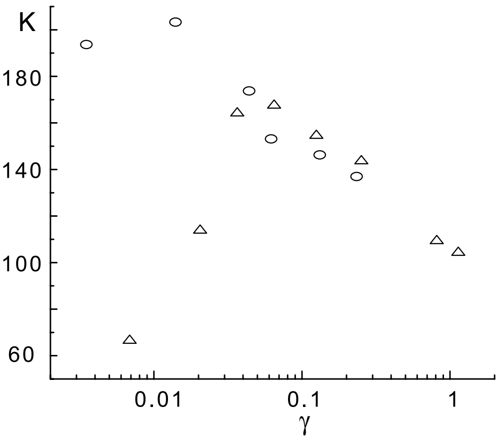

Fig. 7. Dependence of gain factor from parameter γ for input square-wave signal with the amplitude $A = 10$ mV and period 3.3 s. The dimensionless signal frequency is $\omega_s = 0.0113$ (circuit layout 1 (△)) and 6.1×10^{-4} (circuit layout 2 (◯)).

4 Conclusions

To conclude, we show with analog simulations and theoretically that the phenomenon of hypersensitivity to small alternating signals is observed in the system with on-off intermittency under effect of multiplicative controlling noise with finite correlation time $1/\gamma$ and sufficiently large intensity Δ. The gain factor of signal displays a maximum in the optimum range of noise correlation times.

We demonstrate also that for systems with on-off intermittency an universal scaling index of hypersensitivity exists. The condition of hypersensitivity, determined by the value of this index, is the same for different systems with on-off intermittency, and the system specifics reveals itself only in dependence of this factor from system parameters. We hope that the observed phenomenon could be used in new types of detecting devices, and, probably, shed a light on the unique ability of biological systems to detect weak signals in noisy environment.

We are grateful to N.E. Savitskaya for valuable remarks. The work was done within the framework of the RFBR grant N 99-02-17545, and supported by the State program "Physics of quantum and wave processes", subprogram "Statistical physics", project VIII-3, and by State program "Neutron research of condensed state".

References

1. W. Horsthemke, R. Lefever, *Noise-induced phase transitions* (Springer, N.Y., 1984).
2. Z.H. Hou *et al.,* Phys. Rev. Lett. **81**, 2854 (1998).
3. M.O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993).
4. C.R. Doering, W. Horsthemke, J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994).
5. M.M. Millonas, M.I. Dykman, Phys. Lett. A **185**, 65 (1994).
6. R. Bartussek, P. Hänggi, J.R. Kissner, Europhys. Lett. **28**, 459 (1994).
7. R.D. Astumian, F. Moss, Chaos **8**, 533 (1998).
8. K. Wiesenfeld, F. Moss, Nature **373**, 33 (1995).
9. F. Moss, D. Pierson, D. O'Gorman, Int. J. Bifurc. Chaos Appl. Sci. Eng. **4**, 1383 (1994).
10. L. Gammaitoni, P. Hanggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
11. V.S. Anishchenko, A.B. Neiman, F. Moss, L. Shimansky-Geier, Sov. Phys. Usp. **42**, 7 (1999) [Usp. Phys. Nauk **169**, 7 (1999)].
12. S. Kadar, J. Wang, K. Showalter, Nature **391**, 770 (1998).
13. P. Jung, A. Cornell-Bell, K. Madden, F. Moss, J. Neurophysiol. **79**, 1098 (1998).
14. J.F. Lindner *et al.,* Phys. Rev. Lett. **81**, 5048 (1998).
15. P. Castiglione *et al.,* J. Phys. A **31**, 7197 (1998).
16. S.L. Ginzburg, M.A. Pustovoit, Phys. Rev. Lett. **80**, 4840 (1998).
17. S.L. Ginzburg, M.A. Pustovoit, Sov.-JETP Lett. **67**, 627 (1998).
18. O.V. Gerashchenko, S.L. Ginzburg, M.A. Pustovoit, Sov.-JETP Lett. **67**, 997 (1998).
19. O.V. Gerashchenko, S.L. Ginzburg, M.A. Pustovoit, Eur. Phys. J. B **15**, 335 (2000).
20. H. Fujisaka, T. Yamada, Prog. Theor. Phys. **74**, 918 (1985).
21. T. John, R. Stannarius, U. Behn, Phys. Rev. Lett. **83**, 749 (1999).
22. N. Platt, E.A. Spiegel, C. Tresser, Phys. Rev. Lett. **70**, 279 (1993).
23. J.F. Heagy, N. Platt, S.M. Hammel, Phys. Rev. E **49**, 1140 (1994).
24. N. Platt, S.M. Hammel, J.F. Heagy, Phys. Rev. Lett. **72**, 3498 (1994).
25. P.W. Hammer, N. Platt, S.M. Hammel, J.F. Heagy, B.D. Lee, Phys. Rev. Lett. **73**, 1095 (1994).
26. A.S. Pikovsky, Phys. Lett. A **165**, 33 (1992).
27. A.S. Pikovsky, P. Grassberger, J. Phys. A **24**, 4567 (1991).
28. H.L. Yang, E.J. Ding, Phys. Rev. E **54**, 1361 (1996).